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INELASTIC DEFORMATION AND FAILURE ANALYSIS

OF FILAMENT-WOUND COMPOSITE STRUCTURES

Army Research Office Project Number P-25400-EG

Final Report on the Initial Phase of Investigation

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of Filament-Wound Composite Structures

A Report on the Initial Phase of Investigation

1. Introduction

This report summarizes the results of investigation during the initial phase of the project (May to October, 1988). Much of the effort in this period was expended in two areas. First, the existing literature on theoretical and experimental studies of the response, strength and failure behavior of filament-wound tubes was examined and studied. Secondly, a tentative analytical approach was conceived on the basis of available experimental results. The immediate objective of this analytical approach is to describe, in a qualitative manner, the dominant effects associated with the occurrence of the expansion mode and the shear mode of failure, and to provide a mathematical model for the small-strain inelastic deformation accompanying the initiation of such failures. Although these immediate objectives are rather limited in scope, they form the necessary and essential basis for further research to the complex behavior of failure process and inelastic response under large shear deformation. A crucial consideration in the present theory is the areal expansion of the surface of the tube. Its role in determining the mode of failure is postulated and deduced in Section 2 and 3, and some experimental evidence in support of the conclusion is summarized in Section 4. In Section 5, the response of a filament-wound tube under small-strain deformation is examined. The experimentally observed nonlinearity in the response is attributed to the effect of degradation of the elastic moduli of the resin material caused by the formation of microcracks accompanying areal expansion. A model is

proposed in which the degradation of the elastic moduli are represented by a pair of damage parameters. These parameters depend on the prior history of strain. A method of inferring the damage parameters from continuous measurements of axial and circumferential strains corresponding to increasing pressure loading is suggested.

Although the literature on filament-wound tubes is very extensive, in this brief report only a very small number of references were cited. Furthermore, the present report is mainly concerned with outlining the direction of research and the analytical approach to be followed in this project. Certain analytical developments that are at present far from definitive or that have no direct bearing on the main lines of thinking have not been included. Finally, in the present report only the problems associated with a cylindrical tube are considered because this simple geometry lends itself to careful analysis and experiment, and, therefore, offers the best example for preliminary studies. Physical insight, mathematical analysis and correlation of the theory and experiment achieved in the study of cylindrical tubes will eventually be extended and applied to the analysis of more general axisymmetric filament-wound composite structures.

2. Large Deformation of a Cylindrical Surface in a Filament-Wound Tube

We consider a circular cylindrical interior surface element within the diamond-shaped region bounded by two pairs of neighboring fibers, as shown in Figure 1. In the undeformed referential state, each side of the diamond-shaped region has the length L. In the deformed state, the length changes to $L(1+\mathcal{E}_{f})$. Since the parameter \mathcal{E}_{f} is bounded by the fiber failure strain, it is always small. However, during the failure process

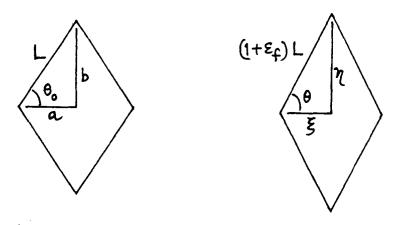
the orientation angle of the fibers may deviate significantly from the initial angle θ_o . The principal stretches λ_1 and λ_2 along the axial and circumferential directions of the filament-wound tube are given, respectively, by

$$\lambda_1 = \frac{\xi}{a} = (i + \varepsilon_f) \frac{\cos \theta}{\cos \theta_0} , \quad \lambda_2 = \frac{\eta}{b} = (i + \varepsilon_f) \frac{\sin \theta}{\sin \theta_0} . \tag{1}$$

The in-plane left and right Cauchy-Green deformation tensors coincide, and their components with respect to the principal directions are

$$\begin{bmatrix} B \\ \approx \end{bmatrix} = \begin{bmatrix} C \\ \approx \end{bmatrix} = \begin{bmatrix} \lambda_1^2 & 0 \\ 0 & \lambda_2^2 \end{bmatrix}$$
 (2)

The force carried by a fiber depends only on the fiber strain \mathcal{E}_{f} , while the average membrane stresses in the resin material are determined by the tensor [B] in the case of large elastic deformation, and by the history of [B] when the deformation is inelastic.



Reference State

Deformed State

Figure 1

We consider an applied external loading which results in a state of uniform axial and circumferential Cauchy stresses (denoted, respectively, by t_1 and t_2 ; they refer to the forces acting on a unit <u>deformed</u> area normal to the axial or the circumferential direction). If the resin material were absent (i.e., if the filament-wound tube consists only of the skeleton of fibers), then the orientation of the fibers under the given state of stress would be given by the angle θ^* such that

$$\left(\frac{T\cos\theta^*}{L\sin\theta^*}\right) / \left(\frac{T\sin\theta^*}{L\cos\theta^*}\right) = \frac{t_1}{t_2},$$

where T is the tension in each fiber. Hence,

$$\tan \theta^* = (t_2/t_1)^{1/2}$$
 (3)

In the case of a closed-end tube subjected to a uniform internal pressure, one has $t_2/t_1=2$. Consequently, $\theta^*=\tan^{-1}\sqrt{2}=54^044^\prime$. This angle is called the optimum winding angle in several existing works on filament-wound tubes. Its optimality is clearly dependent on the specifically assumed loading condition $t_2/t_1=2$.

The presence of the resin material affects the deformation and prevents the fiber net from assuming the orientation angle θ^* . Therefore, the actual orientation angle of the fibers corresponding to the state of loading (t_1, t_2) is between θ_0 and θ^* . Under normal operating conditions, θ should be close to θ_0 . Any significant deviation of θ from θ_0 toward θ^* would be resisted by the shearing strength of the resin material. In any such state of small-strain deformation (i.e., both $\mathcal{E}_{\mathbf{f}}$ and $|\theta-\theta_0|$ are small), an increase in the loading is accompanied by an increase in $\mathcal{E}_{\mathbf{f}}$ and a change of θ away from θ_0 and toward θ^* . Let the increments of $\mathcal{E}_{\mathbf{f}}$ and of

 θ under increased loading be denoted, respectively, by $\xi \mathcal{E}_{f}$ and $\xi \theta$. Then $\xi \mathcal{E}_{f} > 0$ and $(\theta^{*} - \theta_{o}) \xi \theta > 0$, i.e., the algebraic sign of $\theta^{*} - \theta_{o}$ determines that of $\xi \theta$.

If a filament-wound tube is appropriately designed, it should deform in such a way that θ remains close to θ_0 under a wide range of operating loads. Large deformation of the tube, i.e., significant deviation of θ from θ_0 , should occur only when the external load is close to the failure load. In the small-strain deformation states preceding failure, the algebraic sign of $S\theta$ is important because it generally determines the nature or mechanicsm of the failure process, as we shall explain in the following discussion. As mentioned previously, the algebraic sign of $S\theta$ is the same as that of $\theta^* - \theta_0$, and the limiting orientation angle θ^* is determined by the state of loading (Eq. (3)).

3. Shear Failure and Expansion Failure

Deformation of a filament-wound tube may either increase or decrease the surface area of the diamond-shaped region shown in Fig. 1. A significant increase in the area may result in the tensile fracture of the resin material (cohesive failure) or adhesive failure between fiber and matrix. This is associated with experimentally observed progressive whitening and weepage of the tube, formation of droplets and, eventually, fiber breakage and tube rupture. The initiation of this expansion mode of failure may occur at a relatively low level of load. Large strain in the resin material and significant rotation of the fibers do not occur until the external load becomes close to the final failure load.

On the other hand, if the surface area decreases as the external loading increses, then the resin material eventually fails under excessive

shear deformation. There is much less whitening of the tube and no slow formation of droplets. When the shear strength of the resin material is exhausted, the fiber orientation may be significantly different from the initial orientation. Consequently, large deformation of the tube may occur in the failure process, and an accurate prediction of the failure load may require the knowledge of the inelastic resin material behavior under large deformation. Furthermore, failure of the tube in the shear mode may be preceded by extensive delamination since the in-plane areal compression of the resin material may result in buckling of layers.

For the diamond-shaped region shown in Fig. 1, the ratio of the deformed area to the initial area is given by

$$\frac{A}{A_o} = \left(1 + \varepsilon_f\right)^2 \frac{\sin 2\theta}{\sin 2\theta_o} . \tag{4}$$

For a small increase of in the orientation angle of the fibers, the corresponding increase in the area ratio is

$$S\left(\frac{A}{A_0}\right) = \frac{\partial}{\partial \theta} \left(\frac{A}{A_0}\right) S\theta = 2\left(1 + \mathcal{E}_{f}\right)^{2} \frac{\cos 2\theta}{\sin 2\theta_{0}} S\theta. \tag{5}$$

If θ < 45°, then the area increases with increasing θ . The opposite is true if $\tilde{\sigma}$ > 45°. Consequently, there are four possible cases:

- (i) θ < 45 and θ_o < θ^{\bigstar} . In this case the area increase as θ increases toward θ^{\bigstar} . Expansion failure occurs.
- (ii) $\theta > 45^\circ$ and $\theta_o > \theta^*$. In this case the area increases as θ decreases toward θ^* . Expansion failure also occurs.
- (iii) θ < 45° and θ_o > θ^{*} . In this case the area decreases as θ decreases toward θ^{*} . Shear failure occurs.
- (iv) θ >45° and θ_o < θ^{*} . In this case the area decreases as θ increases toward θ^{*} . Shear failure also occurs.

At the initiation of the expansion mode of failure, θ is not significantly different from the initial angle θ_o . Therefore, if the angle θ^* given by Eq. (3) is greater than 45° , then in order to avoid expansional failure one should choose a winding angle θ_o smaller than θ^* . On the other hand, if θ^* is smaller than 45° , then the winding angle should be larger than θ^* . In either case, choosing a winding angle between 45° and θ^* generally ensures the shear mode of failure (except when θ_o is chosen to be very close to 45° or to θ^* , as will be explained below). If θ_o is close to θ^* , then the resin material will not be subjected to excessive shear deformation in the states preceding final failure. The final failure will be initiated by tensile failure of individual fibers.

If either $\theta_0 = \theta^{\times}$ or $\theta = 45^{\circ}$, then (cos 2θ) $\S\theta = 0$ and, consequently, there is no areal expansion due to the rotation of the fibers. However, there is some areal expansion due to the elongation of the fibers (i.e., associated with the strain \S_{\uparrow}). Thus, tubes wound at the ideal winding angle ($\theta_0 = \theta^{\times}$) or at 45° angle will start to fail in the expansion mode. As failure progresses in the 45° filament-wound tube under the closed-end loading condition, θ increases toward θ^{\times} and the failure behavior may subsequently change from the expansion mode to the shear mode.

In the usual production process, a constant pitch is enforced and, as a result, the winding angle θ_0 is smaller toward the interior surface of the tube and larger toward the exterior surface. The deviation from the mean winding angle increases with the thickness of the tube and decreases with its radius. This three-dimensional effect influences the accuracy of the preceding analysis. It also implies that, for whatever choice of the helical pitch, a certain amount of in-plane shear deformation necessarily occurs in the resin material and this shear deformation varies in the

thickness direction of the tube. If the external loading has a fixed ratio t_2/t_1 so that θ^{\bigstar} remains constant throught the loading process, then the winding pitch should be chosen so that the winding angles at the interior and exterior surfaces of the tube are both within the range between 45° and θ^{\bigstar} , and that one of the winding angle should preferably be equal to θ^{\bigstar} .

4. Experimental Evidences

The preceding theoretical predictions are supported by numerous experimental results available from the existing literature. Hull, Legg and Spencer (1978) tested glass/polyester filament wound tubes with the "ideal" winding angle (54°44′) under both closed-end (t_2 / t_1 = 2) and unrestrained-end (axial load = 0) conditions. They referred to the first case as Mode 2 loading and the second case as Mode 3 loading. Since the fiber orientation tends to stay at the "ideal" angle under the closed-end loading, the surface area expands because of fiber extensional strain \mathcal{E}_{f} rather than because of fiber rotation. Hence the tube fails under the expansion mode. In the unrestrained-end loading case, the fibers tend to orient toward the circumferential direction (θ^* = 90°). The surface area decreases in the deformation process, and the tube fails in the shear mode. The observed failure phenomena have also been confirmed by examination of photo-micrographs (Jones and Hull, 1979).

More extensive testing of pipes wound at 35, 45, 65 and 75 degree angles (Spencer and Hull, 1978) further supports the theoretical predictions of the preceding two sections. Under both closed-end and unrestrained-end conditions, the 35° tubes clearly fail under the expansion mode. Weepage and whitening begins at a very low level of loading. The

orientation angle of the fibers increases from 35° and eventually reaches 50° in the state immediately preceding final fracture. For the tubes wound at 45° angle, the area increase due to increase in θ is zero initially (because $\cos 2\theta = 0$ in Eq. (5)) and small subsequently. The initial failure of the tube is also that of the expansion mode because the surface area increases with fiber elongation. However, as θ increases and deviates significantly from 45° , the decrease in the area ratio due to the increase of θ (as given by Eq. (5)) dominates over the increase caused by fiber elongation. Hence the failure process should change into that of the shear mode. Spencer and Hull observed less extensive whitening and reduced over-all breakdown of the tube for tubes wound at 45° angle. Furthermore, under the unrestrained-end loading case (i.e., axial load = 0), local delamination were observed which suggested the reduction of surface area accompanying the shear mode of failure.

The 65° and 75° tubes under the closed-end loading condition showed very extensive whitening after weepage and generated creaking noise at the instant of failure. The phenomena are clearly associated with the expansion mode of failure as the surface area increases when the fiber orientation angle decreases toward the limiting angle $\theta^*=55^\circ44'$. On the other hand, the same tubes (especially the tubes wound at 75° angle) showed much less whitening under the unrestrained-end loading condition. The onse⁺ of weepage occurred suddenly with the formation of jets of liquid through the tube wall as the tube buckled before final failure. The failure process clearly belonged to the shear mode.

5. Nonlinear Response of the Tube at Small Strain and the Initiation of Failure

For tubes wound at the ideal winding angle (54°44') and loaded under the closed-end condition, Hull, Legg and Spencer (1978) reported significant deviations from linear relationship between the pressure load and the axial strain of the tube, at a low level of loading such that both the axial and circumferential strains are very small (see Figure 5 in their paper). The nonlinear stress-strain relation began at a load level corresponding to the occurrence of weepage, when the axial strain of the tube was smaller than 0.2 percent. This nonlinear response cannot be attributed to the plastic deformation of the resin material, because the shear deformation of the resin material is small. A tentative conclusion of the present investigation is that the nonlinear response of the tube at small strain is the result of continuously distributed damage caused by the formation of microcracks when the resin material is subjected to areal expansion. Damage accumulation manifests itself not only in the weepage of the tube, but also in the degradation of the elastic moduli of the resin material. The white streaks in the resin material are observed to be parallel to one family of fibers. This suggests that the degradation of the elastic moduli of the resin material is direction-dependent. The resin material loses its elastic isotropy as microcracks continue to form along a preferred direction. At a high level of pressure load, the resin material suffers significant damage and loses much of its stiffness. The membrane forces in the tube are essentially carried only by the skeleton of fibers and, consequently, the axial strain becomes very close to the circumferential strain. The burst strain is reported to be about 1.6 percent. At this level of failure strain, the shear deformation of the resin material is not large and hence the almost complete loss of resin stiffness cannot be attributed primarily to plasticity effects.

Since the entire failure process is strongly dependent on the conditions at the initiation of failure, a rational investigation of the failure process must begin with a careful analysis of the inelastic deformation of the tube at small strain. A complete solution of this difficult problem has not been achieved at this stage of investigation. However, a description of the method of the present approach will be given in the following.

Let the Young's modulus and the cross-sectional area of a fiber be denoted, respectively, by \mathbf{E}_f and \mathbf{A}_f . Then the tension in the fiber is given by

If V_f denotes the fiber volume fraction, and \mathcal{T}_1^M and \mathcal{T}_2^M stand for the average stresses in the resin material along the axial and circumferential directions, respectively. Then the equilibrium equations in the two principal membrane directions are given by

$$(E_f A_f/hL) \mathcal{E}_f + (1-V_f) (1+\mathcal{E}_f) \sigma_1^m \tan \theta = (1+\mathcal{E}_f) t_1 \tan \theta,$$

$$(E_f A_f/hL) \mathcal{E}_f + (1-V_f) (1+\mathcal{E}_f) \sigma_2^m \cot \theta = (1+\mathcal{E}_f) t_2 \cot \theta,$$

$$(6)$$

where h is the spacing in the thickness direction between two adjacent layers of fibers. Under a given type of loading, say, closed-end loading condition, the two principal stretches λ_1 and λ_2 are simultaneously measured for continuously increasing t_2 . At any instant of the loading process, the fiber strain $\mathcal{E}_{\mathbf{f}}$ and the orientation angle θ may be calculated from

$$\mathcal{E}_{f} = \left(\lambda_{1}^{2} \cos^{2} \theta_{0} + \lambda_{2}^{2} \sin^{2} \theta_{0}\right)^{1/2} - 1,$$

$$\tan \theta = \frac{\lambda_{2} \sin \theta_{0}}{\lambda_{1} \cos \theta_{0}}.$$

The preceding two expressions follow from Eq. (1). The system of equations (6) may be solved for the resin average stresses $\mathcal{O}_1^{\mathsf{M}}$ and $\mathcal{O}_2^{\mathsf{M}}$. Thus, the test data provide the association of a set of measured principal stretches (λ_1,λ_2) with a corresponding set of calculated average resin stresses $(\mathcal{O}_1^{\mathsf{M}},\mathcal{O}_2^{\mathsf{M}})$. The relationship provides the information on stiffness degradation under the particular type of loading condition.

While the preceding formulation is valid regardless of the magnitude of deformation, in the case of small strains more specific results may be obtained. Let

$$\varepsilon_1 = \lambda_1 - 1$$
, $\varepsilon_2 = \lambda_2 - 1$

denote the axial and circumferential strains when both are small. If we assume that damage in the resin material causes degradation in the extensional moduli along the two principal direction without appreciably affecting the Poisson's ratio (this assumption is tentative and its validity should be critically examined on this basis of experimental data), then the relation between the average resin stresses and the average resin strains (the latter considered to be identical to \mathcal{E}_1 and \mathcal{E}_2 , respectively) is given by

$$\mathcal{E}_{1} = (\sigma_{1}^{\text{M}} - \nu \sigma_{2}^{\text{M}}) / (S_{1} E_{m}), \quad \mathcal{E}_{2} = (\sigma_{2}^{\text{M}} - \nu \sigma_{1}^{\text{M}}) / (S_{2} E_{m})$$

where $\mathcal V$ is the Poisson's ratio of the resin material (assumed to be unaffected by resin cracking) and S_1 and S_2 are damage parameters characterizing the degradation of the elastic moduli in the axial and circumferential directions, respectively. Since the values of $\mathcal S_1^{\mathsf M}$ and $\mathcal S_2^{\mathsf M}$ may be calculated in terms of the measured average strains $\mathcal S_1$ and $\mathcal S_2$, the damage parameters S_1 and S_2 may be found as function of resin strains.

It may be realistic to expect that the two damage parameters depend only on the areal expansion ratio $\lambda_1 \lambda_2$, which in the case of small strains reduces to 1 + ξ_1 + ξ_2 .

Similar analyses may be performed for test data obtained from tubes wound at different winding angles and subjected to different loading conditions (for example, the unrestrained end condition, $t_1=0$). The damage parameters determined from the different experimental conditions may be correlated to suggest a consistent mathematical model of damage in the resin material and this model may be used to predict the degradation of resin moduli and to establish the inelastic consitutive equations for other winding angles and loading conditions. Such an analysis may provide the basis for a theory of failure initiation in the filament-wound tube. The analysis of the subsequent failure process and the ultimate failure load will generally require the consideration of large deformation of the resin material.

It should be pointed out that, in the regime of small strains, linear elastic response must be expected if damage of the resin material were not taken into account. However, the linear response is contrary to the experimental evidence since the latter clearly indicates nonlinear relationship between the pressure loading and the axial strain even when both strain components are small (see, in addition to previously cited work of Hull, Legg and Spencer, the test data of Clodfelter, 1980, under the unrestrained-end loading condition, particularly Figures 4, 13 and 14 and the data of Vandiver, 1985). The introduction of the damage parameters S_1 and S_2 results in nonlinearity in the response. Furthermore, experimental results also show significant departure of the stress-strain curve in the unloading process from the corresponding curve in the loading

process (Clodfelter, 1975, Figs. A-1 to A-18 and A-32 to A-36). This provides further evidence of the degradation of the elastic moduli of the resin material under relatively small levels of strain.

Although the formation of densely distributed transverse cracks in the resin material and its degradation effect on the elastic moduli have been noticed in cross-ply composite laminates (see, for example, Parviai and Bailey, 1978), the various theories of failure initiation existing in the literature on filament-wound tubes (for example, work in Germany by Puck, Forster and their schools as reviewed in Greenwood, 1977) are generally based on linearly elastic constituive relations without consideration of this effect. Such theories cannot explain the nonlinear response evident in small-strain test results. This shortcoming is not shared by the present analytical approach. When this approach is fully developed in the subsequent phases of the research and when the theoretical predictions are correlated with the experimental data (e.g., Clodfelter, 1975 and Vandiver, 1985), it may suggest appropriate mathematical models capable of providing reliable and consistent predictions of failure initiation and development. The results will form the basis for the analysis of inelastic deformation accompanying the final failure, the identification of the various mechanisms operating under different types of failure processes, and the estimation of the ultimate failure loads.

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